# First Order System Types

## 1. Introduction:

First order systems are, by definition, systems whose input-output relationship is a first order differential equation. A first order differential equation contains a first order derivative but no derivative higher than first order – the order of a differential equation is the order of the highest order derivative present in the equation.

First order systems contain a single energy storage element. In general, the order of the input-output differential equation will be the same as the number of independent energy storage elements in the system. Independent energy storage cannot be combined with other energy storage elements to form a single equivalent energy storage element. For example, we previously learned that two capacitors in parallel can be modeled as a single equivalent capacitor – therefore, a parallel combination of two capacitors forms a single independent energy storage element.

First order systems are an extremely important class of systems. Many practical systems are first order; for example, the mass-damper system and the mass heating system are both first order systems. Higher order systems can often be approximated as first order systems to a reasonable degree of accuracy if they have a dominant first order mode.

## 2. First Order System Model

The first order system has only one pole as shown

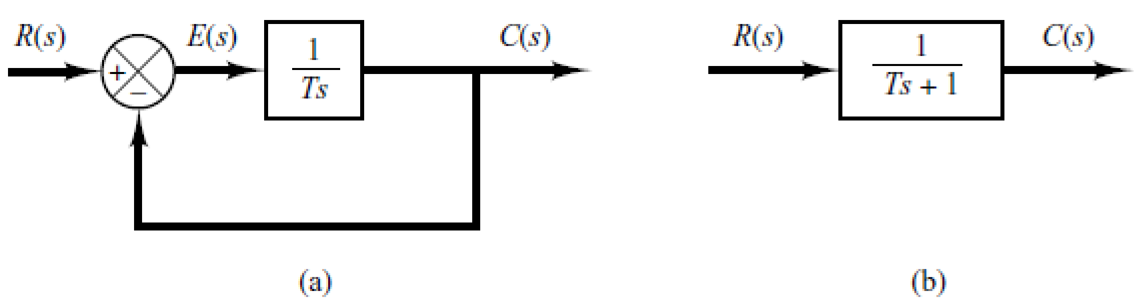


Figure 1: (a) Block Diagran of a first-order system; (b) Simplifed block Diagram

$$\frac{C(s)}{R(s)} = K\frac{1}{Ts+1} \tag{1}$$

- Where $\boldsymbol{K}$ is the DC Gain and $\boldsymbol{T}$ is the time constant of the system.
- Time Constant is a measure of how quickly a $1^{st}$ order system response to a unit step input.
- DC gain of the system ration between the input signal and the steady state value of output.

Example 1:

For the first order system given below

$$G(s) = \frac{10}{3s+1}$$

- DC gain (K) is equal 10
- Time Constant (T) is equal 3

Example 2:

For the first order system given below

$$G(s) = \frac{3}{s+5} = \frac{\frac{3}{5}}{\frac{1}{5}s+1}$$

- DC gain (K) is equal $\frac{3}{5}$
- Time Constant (T) is equal $\frac{1}{5}$

## 2.1 Impulse Response of $1^{st}$ Order System

Consider the following $1^{st}$ order system in figure 2

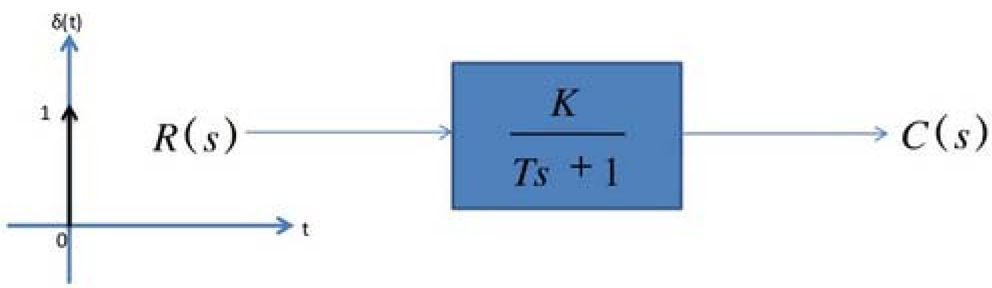


Figure 2: Impulse response of first order system

$$R(s) = \delta(s) = 1$$
$$C(s) = \frac{K}{Ts+1}$$

In order to represent the response of the system in time domain we need to compute the inverse Laplace transform of the above equation, we have

$$c(t) = \frac{K}{T}e^{-\frac{t}{T}} \tag{2}$$

Example 3:

For the first order system given below

$$G(s) = \frac{3}{2s+1}$$

The impulse response is shown in figure 3

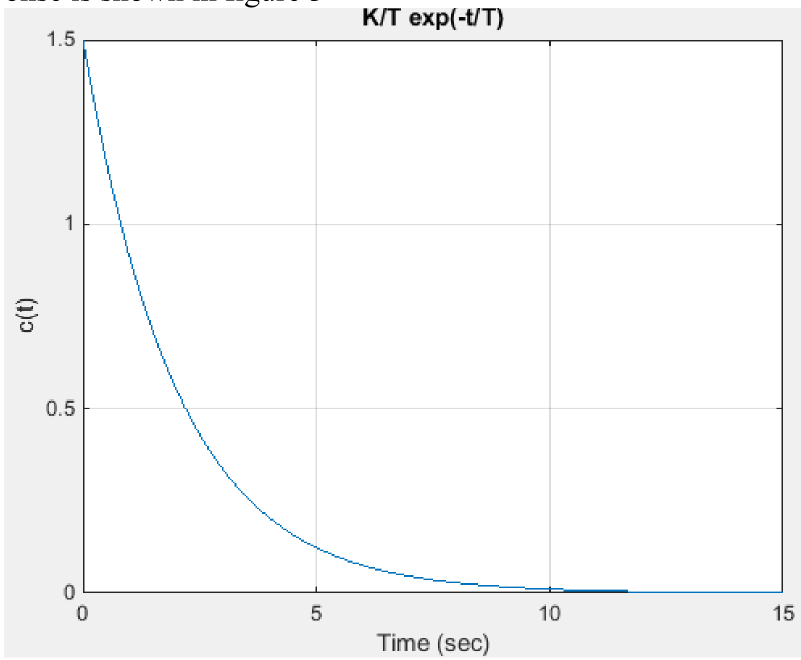


Figure 3: Impulse response of example 3

## 2.2 Ramp Response of $1^{st}$ Order System

Consider the first order system in figure 1.

$$R(s) = \frac{1}{s^2}$$

$$C(s) = \frac{K}{Ts+1}\frac{1}{s^2}$$

In order to represent the response of the system in time domain we need to compute the inverse Laplace transform of the above equation, we have

$$c(t) = Kt - T + Te^{-\frac{t}{T}} \tag{3}$$

Example 4:

For the first order system given below

$$G(s) = \frac{K}{Ts + 1}$$

1) The ramp response of system, if $K = 1 \; and \; T = 1$ is shown in figure 4
2) The ramp response of system, if $K = 1 \; and \; T = 3$ is shown in figure 5

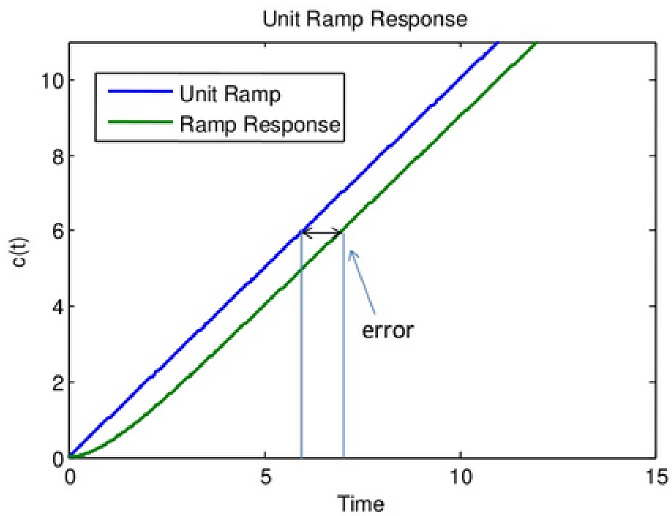


Figure 4: The ramp response for case 1, example 4

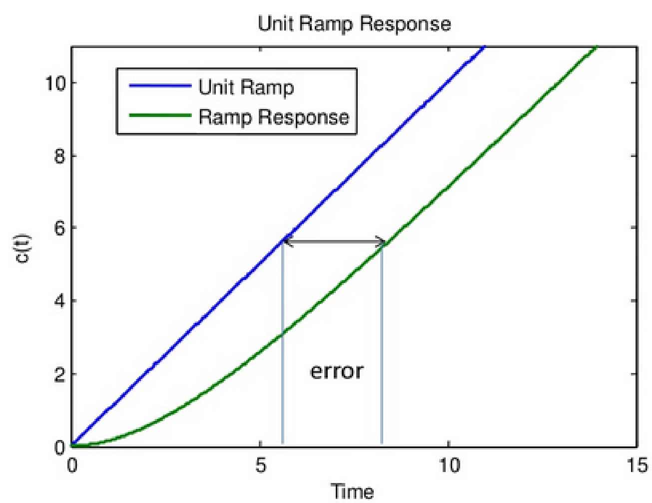


Figure 5: The ramp response for case 2, example 4

## 2.3 Step Response of $1^{st}$ Order System

Consider the first order system in figure 1.

$$R(s) = \frac{1}{s}$$
$$C(s) = \frac{K}{Ts+1}\frac{1}{s}$$

In order to represent the response of the system in time domain we need to compute the inverse Laplace transform of the above equation, we have

$$c(t) = Ku(t) - e^{-\frac{t}{T}} \tag{4}$$

1) Where $u(t) = 1$

$$c(t) = K - e^{-\frac{t}{T}} \tag{5}$$

2) Where $t = T$

$$c(t) = K - e^{-1} = 0.632K \tag{6}$$

- For example, assume $\boldsymbol{K = 10, T = 1.5s}$

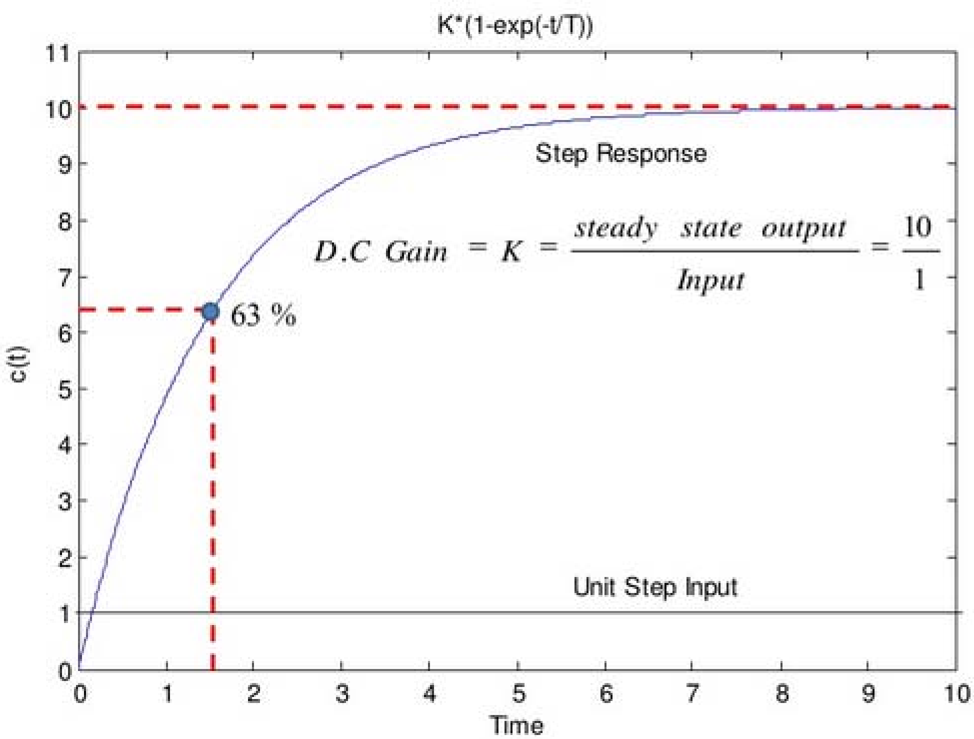


Figure 6: The step response specification of first order system

The step response of the first order system takes five time constants to reach its final value.

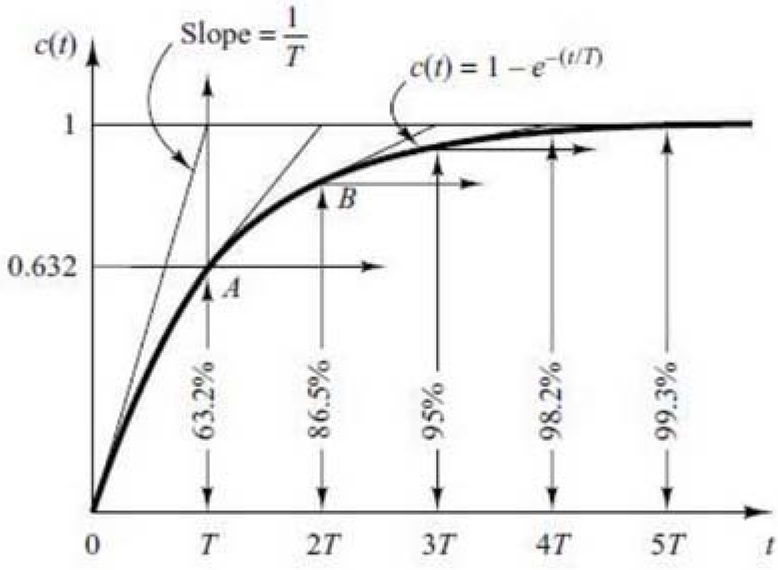


Figure 7: Step response

- $\boldsymbol{K = 10, T = 1, 3, 5, 7\ s}$

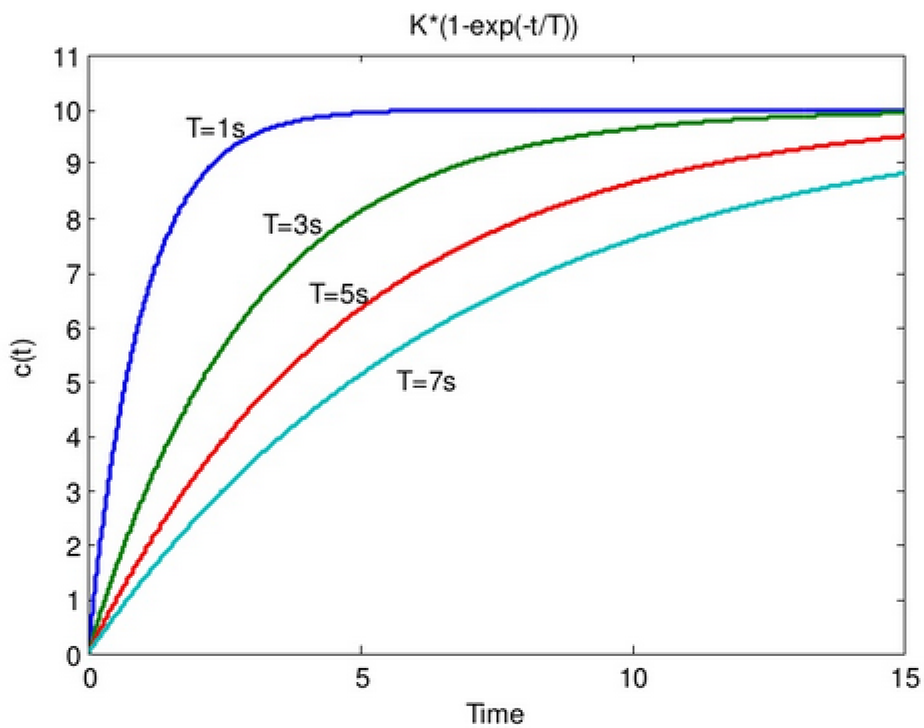


Figure 8: The step response at different value of T

- $\boldsymbol{K = 1, 3, 5, 10\ , T = 1\ s}$

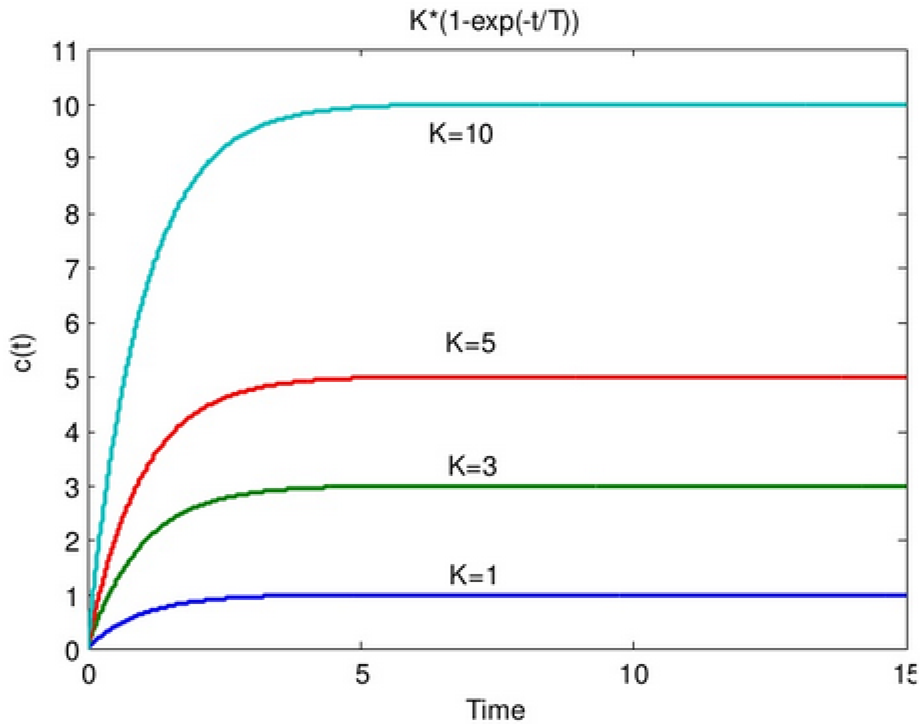

Figure 9: The step response at different value of K

## 3. First Order System with a Zero

The transfer function of the first order system with a zero can be represented using the form in equation (7)

$$G(s) = \frac{C(s)}{R(s)} = \frac{K(1+\alpha s)}{Ts+1} \tag{7}$$

- Zero of the system lie at $s = -1/\alpha$ and pole at $s = -1/T$
- Step response of the system would be:

$$C(s) = K\frac{1}{s}\frac{(1+\alpha s)}{Ts+1}$$

The Laplace inverse transfer is

$$c(t) = K + \frac{K}{T}(\alpha - T)\, e^{-\frac{t}{T}} \tag{8}$$

- **Case 1:** $\boldsymbol{T > \alpha}$

The shape of the step response is approximately same (with offset added by zero)

<u>Example 5:</u>

Consider the first order system given by

$$\frac{C(s)}{R(s)} = \frac{10\,(1+2s)}{3s+1}$$

In this system:

1. $K = 10$
2. $T = 3$
3. $\alpha = 2$

$$c(t) = 10 + \frac{10}{3}(2-3)e^{-t/3}$$

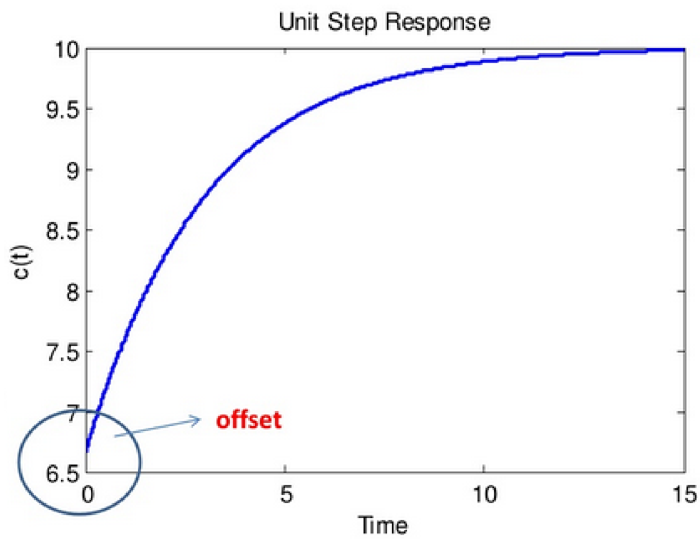

Figure 10: the step response of case 1 $\boldsymbol{T > \alpha}$

$$offset = K + \frac{K}{T}(\alpha - T)$$

- **Case 2:** $\boldsymbol{T < \alpha}$

<u>Example 6:</u>

Consider the first order system given by

$$\frac{C(s)}{R(s)} = \frac{10\,(1+2s)}{1.5s+1}$$

In this system:

1. $K = 10$
2. $T = 1.5$
3. $\alpha = 2$

$$c(t) = 10 + \frac{10}{1.5}(2-1)e^{-t/1.5}$$

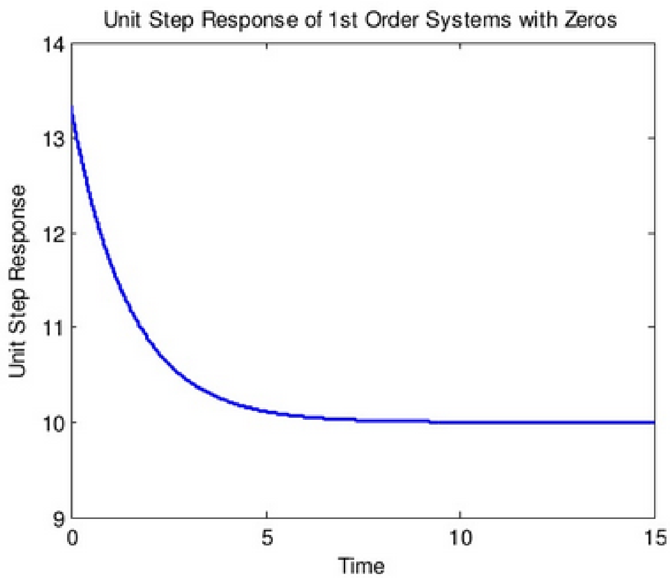


Figure 11: The step response for case 2 $\boldsymbol{T < \alpha}$

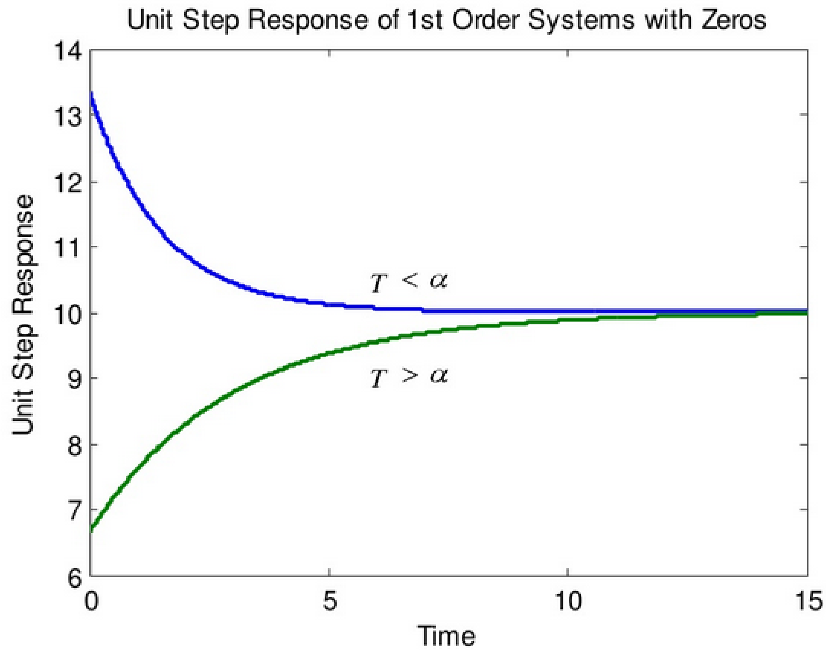


Figure 12: The step response of first order system with zero

## 4. First Order System with Delays

The first order system with delay time can have the following transfer function

$$\frac{C(s)}{R(s)} = \frac{K}{Ts+1}e^{-st_d} \tag{9}$$

Where
$t_d$ : *is the delay time*

The step response of this type of system is shown in figure 13

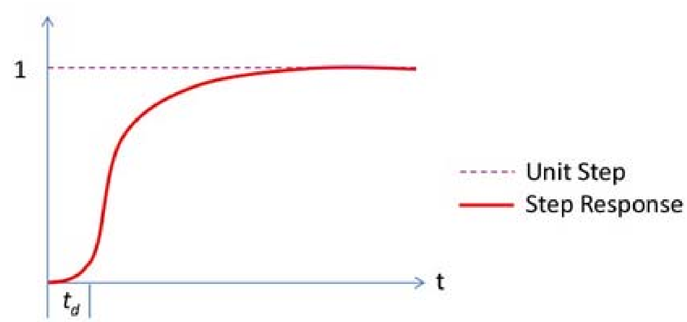


Figure 13: the step response of first order system with delay time

Example 6:

Consider the following first order system

$$G(s) = \frac{10}{s+1}e^{-2s}$$

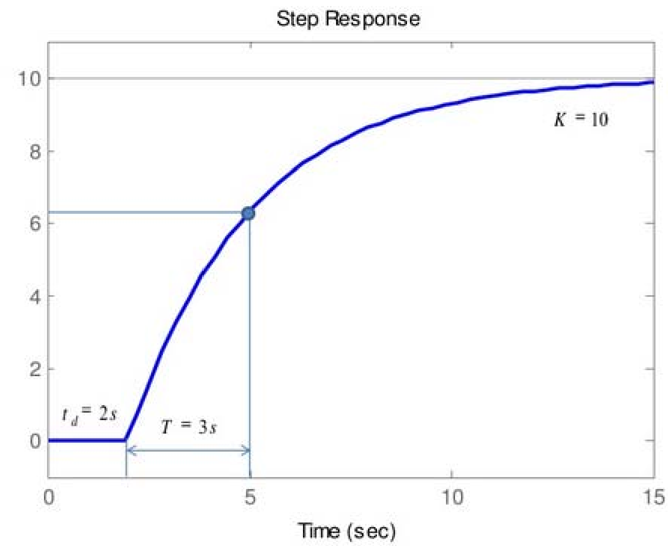

Figure 14: step response for system in Example 6

## 5. System Identification of Transfer Function of $1^{st}$ Order Systems

The field of system identification uses statistical methods to build mathematical models of dynamical systems from measured data. System identification also includes the optimal design of experiments for efficiently generating informative data for fitting such models as well as model reduction.

A much more common approach is therefore to start from measurements of the behavior of the system and the external influences (inputs to the system) and try to determine a mathematical relation between them without going into the details of what is actually happening inside the system. This approach is called system identification. Two types of models are common in the field of system identification:

1) **Grey box Model**: although the peculiarities of what is going on inside the system are not entirely known, a certain model based on both insight into the system and experimental data is constructed. This model does however still have a number of unknown free parameters which can be estimated using system identification.

   One example, uses the Monod saturation model for microbial growth. The model contains a simple hyperbolic relationship between substrate concentration and growth rate, but this can be justified by molecules binding to a substrate without going into detail on the types of molecules or types of binding. Grey box modeling is also known as semi-physical modeling.

2) **Black box Model**: No prior model is available. Most system identification algorithms are of this type.

In science, computing, and engineering, a black box is a device, system or object which can be viewed in terms of its inputs and outputs (or transfer characteristics), without any knowledge of its internal workings. Its implementation is "opaque" (black). Almost anything might be referred to as a black box: a transistor, algorithm, or the human brain.


Figure 15: Black Box

## 5.1 System Identification of First order system

- Often it is not possible or practical to obtain a system's transfer function analytically.
- Perhaps the system is closed, and the component parts are not easily identifiable.
- The system's step response can lead to a representation even though the inner construction is not known.
- With a step input, we can measure the time constant and the steady-state value, from which the transfer function can be calculated.
- If we can identify T and K from laboratory testing we can obtain the transfer function of the system.

Example 7:

Assume the unit step response given in figure 16;

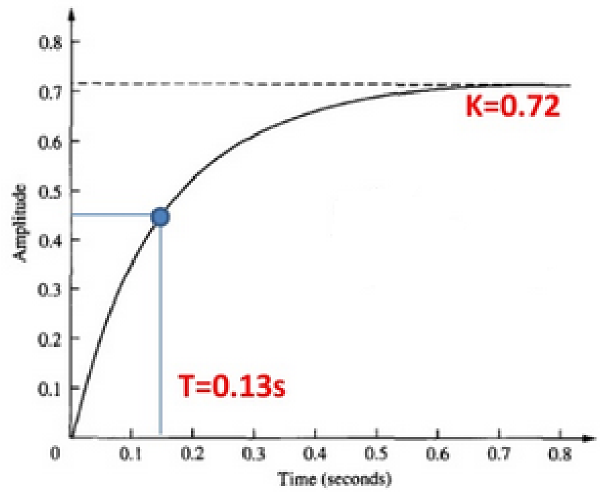


Figure 16: Unit step Response

- From the response, we can measure the ***time constant T,*** that is the time constant for the amplitude to reach 63% of its final value.

Since the final value is about 0.72 the time constant is evaluated for the curve reaches

$0.63 \times 0.72 = 0.45$, then $T = 0.13$

- $K$ is simply the steady state value.
- The transfer function is obtained as:
  $$\frac{C(s)}{R(s)} = \frac{0.72}{0.13s + 1} = \frac{5.5}{s + 7.7}$$

# 6. Simulation of First order system using Simulink

In this section we study a open loop and closed loop system for case a first order system with delay and show the parameter of first order system.

**Note:** Maybe there more than blocks representation but we discuss and use the most model simulate the practical experiments as shown in the following example

Example 8:

Consider the first order system with delay given by the following transfer function

$$G(s) = \frac{2}{3s+1}e^{-1s}$$

As shown in the previous section

$K = 2 \quad (\,DC\ gain)$
$T = 3 \ \ (Time\ Constant)$
$t_d = 1 \ \ (delay\ time)$

We can simulate this system in Simulink using the basic block diagrams (Transfer Fcn, gain , sum and Transport Delay)

- **Open Loop System**

We assume the system in unit step input.

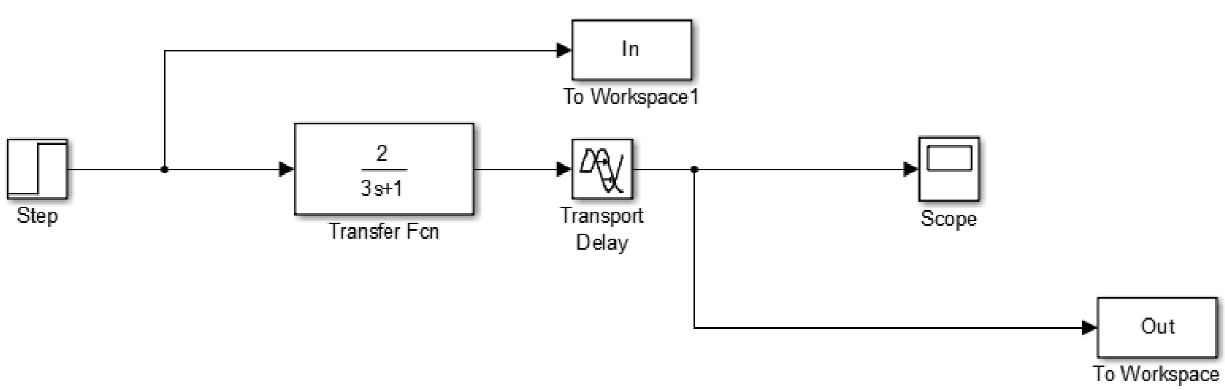


Figure 17: Open loop of system in example 8

The result of step response and the parameter is shown in figure 18

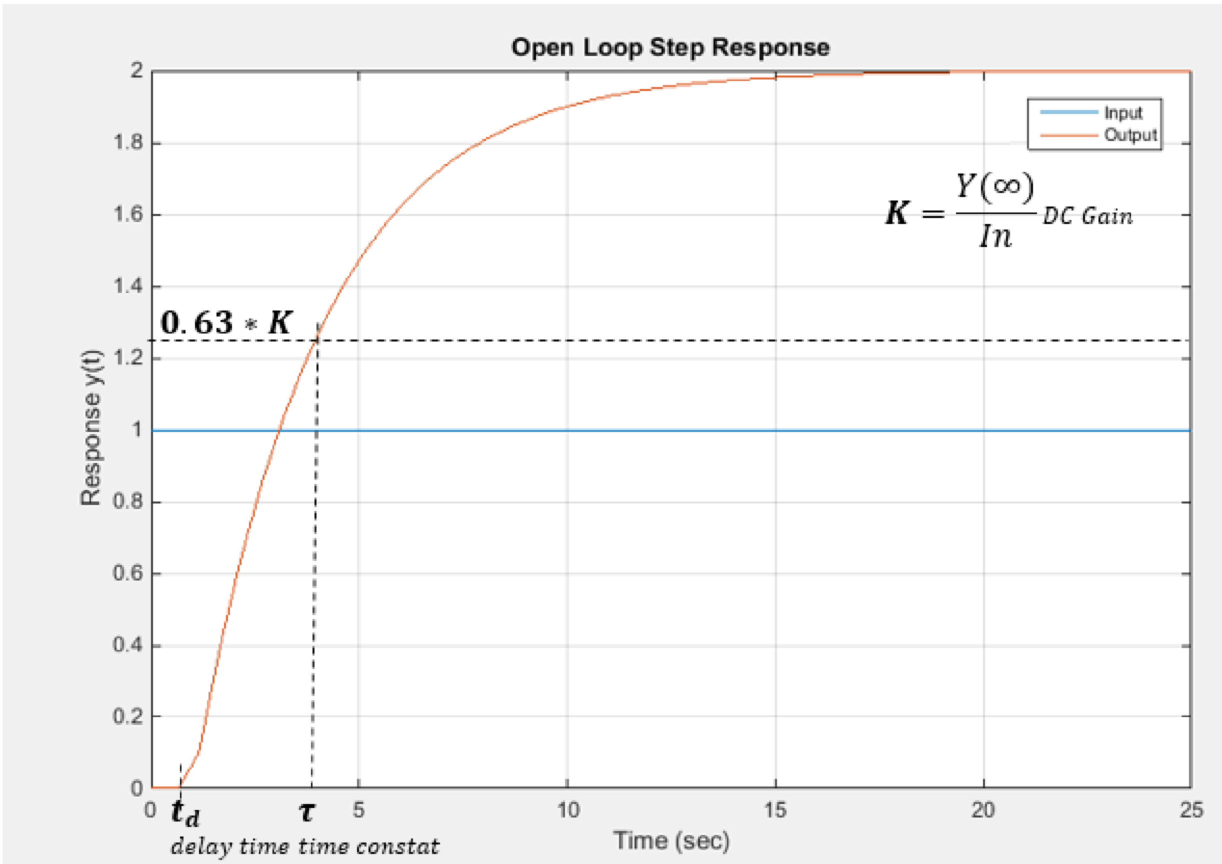


Figure 18: Open loop Response

So if the experimental data for first order system is known we can obtain the transfer function by determine the parameter $(K, \tau, t_d)$ as discussed in the system identification example.

- **Closed loop Response**

The block diagram representation of closed loop system is shown in the figure 19

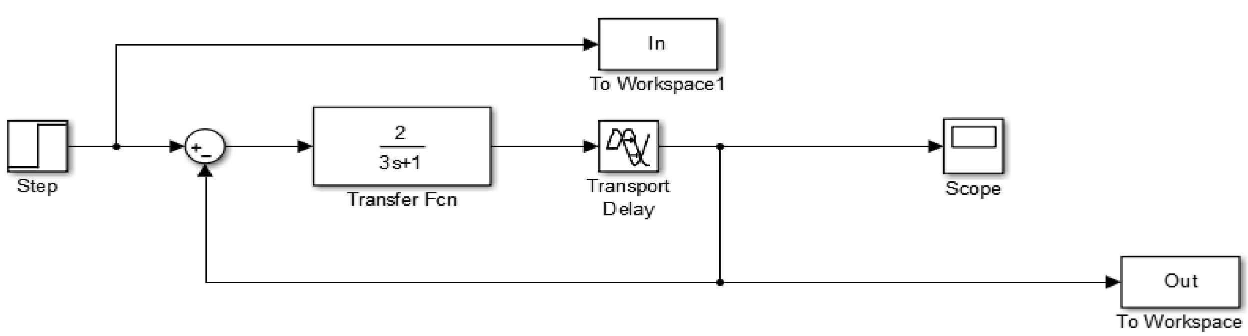


Figure 19: Closed Loop Bock Diagram

The step response is shown in figure 20

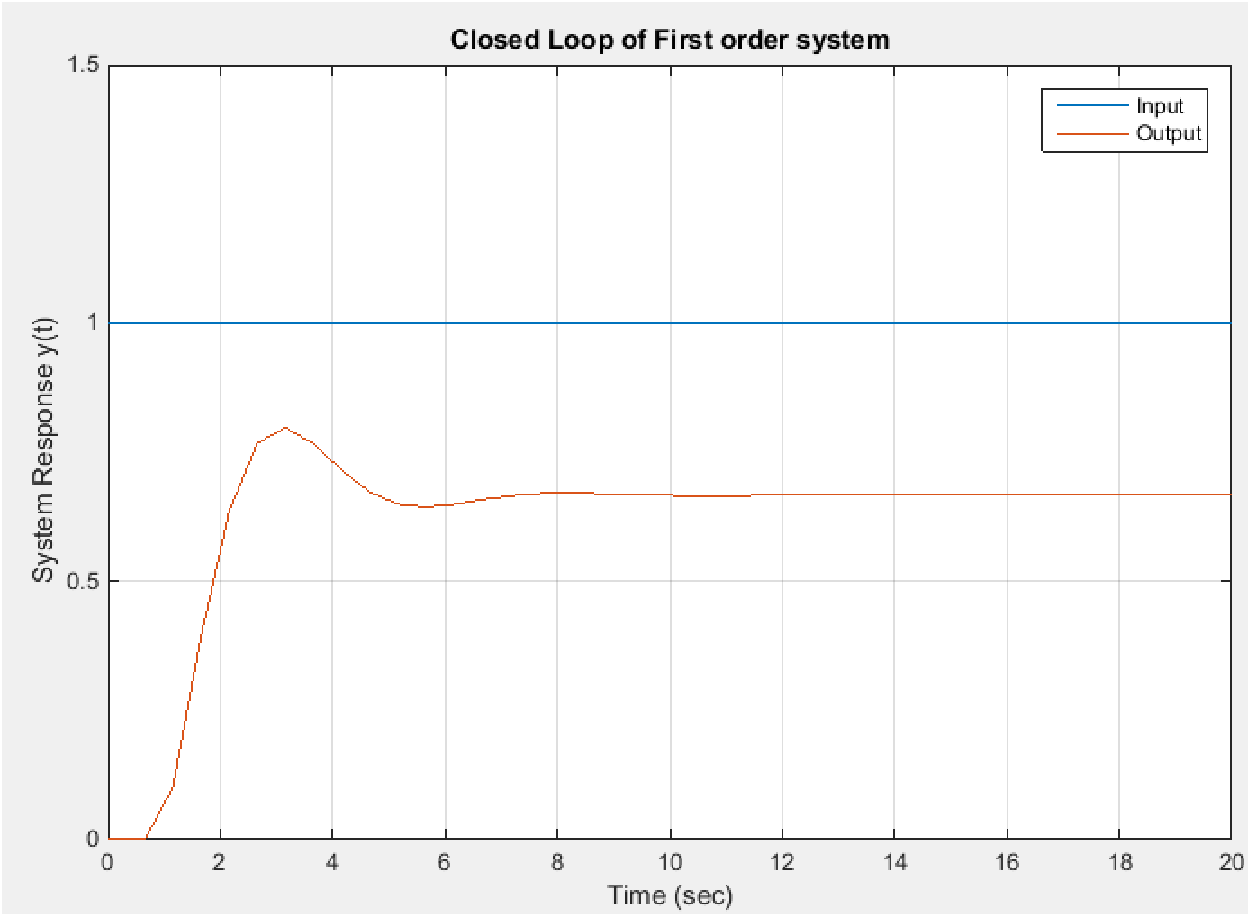


Figure 20: Step Response of closed loop

Depend on the two responses of system, we observe to determine the transfer function of the system we must get the parameter from the open loop characteristic because in this case we obtain the response and the behavior of the system without any modification or addition effect.